GRADUIERTENKOLLEG
Identification in Mathematical Models:
Synergy of Stochastic and Numerical Methods

Workshop on Statistical Inverse Problems

University of Göttingen, Germany
March 23-25, 2006

Talks

Applications of Bayesian hypermodels

Erkki Somersalo
Helsinki University of Technology
Institute of Mathematics
P.O.Box 1100
FI-02015 TKK
Finland
esomersa@math.hut.fi

Bayesian hypermodels provide a flexible tool for defining priors that are based on qualitative description of what we believe about the solution of an inverse problem. In this talk, we consider inverse problems in signal analysis and image processing and demonstrate the use of hyperpriors in these contexts.

The Large Binocular Telescope: A Laboratory for developing Image Reconstruction in Astronomy

Mario Bertero
University of Genova
DISI - Dipartimento di Informatica e Scienze dell'Informazione
Via Dodecaneso, 35
16146 Genova
Italy
bertero@disi.unige.it

The Large Binocular Telescope (LBT), under construction on the top of Mount Graham (Arizona), will consists of two 8.4m mirrors on a common mount, with a distance of 14.4m between their centers. It will be equipped with a Fizeau interferometer, developed by a consortium of German and Italian institutions and denoted LINC-NIRVANA. For a given astronomical target LBT LINC-NIRVANA will provide a set of different images corresponding to different orientations of the baseline; the problem is to reconstruct a unique high-resolution image, possibly with the resolution of a 22.8m mirror. Therefore LBT LINC-NIRVANA will require routinely the use of image reconstruction methods.

In this lecture, after a brief description of LBT and of the main features of its images, the classical maximum likelihood (ML) approach to image reconstruction, based on an accurate statistical description of the noise corrupting images detected by a CCD camera, is discussed as well as the approximations leading to problems such as the minimization of the Csiszar I-divergence or of the least-squares functional. Regularization can be provided by a Bayesian approach, different priors being used for describing different classes of astronomical targets with specific features. Therefore the computation of the maximum a posteriori (MAP) estimates also leads to a wide class of minimization problems, with additional constraints such as non-negativity, flux conservation, etc.

A general approach to the design of iterative methods converging to constrained ML or MAP estimates is presented and specific examples are given. Numerical results on the reconstruction of different classes of astronomical objects are discussed from the point of view of accuracy and efficiency.

A class of stochastic inverse problems: curves warping

Jean-Michel Loubes
Université de Montpellier 2, Département de mathématiques
Equipe de Probabilités et Statistique/ UMR 7641
place E. Bataillon, bat 9, bureau 412
34000 Montpellier
France
loubes@math.univ-montp2.fr

The aim of this work is to tackle stochastic inverse problems in biology. Namely, the concentration of a medicine in blood is well represented by a time dependent function $t \in \mathbb{R}_+ \mapsto q_s(t) \in \mathbb{R}_+$, often referred as the "pharmacokinetic". The structure of this function is known and usually arises from mechanistic structural models. The s parameter makes the kinetic individual dependent, which corresponds to the biological specificity of each individual. In population pharmacokinetics approaches, the individuals are independent, and their s parameters are thus i.i.d. and *unknown*. Indeed, everyone reacts structurally the same way to a specific medicine, but there are biological differences from one individual to another, which are expressed throughout the parameter. The main interests in population pharmacokinetics is the stochastic inverse problem which consists in the estimation of the common law of the unknown s from the observations of sparse values of the kinetics q_s.

This problem is closely related to the issue of finding the strutural pattern in a set of curves and studying the differences between the realization of the experiments. Such issues are common in functional data analysis and data mining. Here we propose a methodology to estimate, in an inverse problem framework, the unknown pattern, the warping parameters and their density in the random deformation case.

Fréchet differentiation of functions of operators with application in functional data analysis

Frits Ruymgaart
Department of Mathematics and Statistics
Texas Tech University
Lubbock, TX 79409
United States of America
h.ruymgaart@ttu.edu

This paper deals on the one hand with some statistical problems regarding functional data analysis and on the other with a mathematical tool from perturbation theory for these problems, that might be of independent interest. The statistical area of application includes more specifically inverse problems with random operators that occur in econometrics, and limiting distributions of functional canonical correlations. The mathematical tool to be derived is a Fréchet derivative of an analytic function (in the sense of functional calculus) of a compact, nonnegative Hermitian operator tangentially to the space of all compact Hermitian operators. This Fréchet derivative enables one to obtain an approximation of a function (a regularized inverse, for instance) of a (random) perturbation of an operator, and the asymptotic distribution of a function of a random operator (for instance a covariance operator).

Inverse Problems: Strategies to counter noise which exhibits bad behavior

(Joint research with Sergei Pereverzyev and Axel Munk)

Frank Bauer
Institute for Numerical and Applied Mathematics
University of Göttingen
Department of Mathematics
Lotzestr. 16-18
37083 Göttingen
Germany
fbauer@math.uni-goettingen.de

Most of the time inverse problems are investigated in a kind of well-protected environment. With respect to a certain noise model it is shown that regularization algorithms and parameter choice schemes work. Common assumptions are white noise or even deterministic noise models. One of the recently investigated parameter choice schemes has been the (Lepskij-type) balancing principle which exhibits a surprising stability in numerical tests.

We investigate two separate situations using this parameter choice method. In the first we will consider the case of heavy tail noise models and different kinds of norms. In the second we will propose a multi-parameter regularization scheme which seems to be very stable towards misspecifications of the covariance operator of the noise.

Regularization Algorithms in Learning Theory

(Joint research with Frank Bauer and Lorenzo Rosasco)

Sergei Pereverzyev
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
Austrian Academy of Sciences
Altenbergerstrae 69
A-4040 Linz
Austria
sergei.pereverzyev@oeaw.ac.at

We discuss a relation between Learning Theory and Regularization of linear ill-posed problems. It is well known that Tikhonov regularization can be profitably used in the context of supervised learning, where it is usually goes under the name of regularized least-squares algorithm. Moreover, the gradient descent learning algorithm has been studied recently, which is an analog of Landweber regularization scheme. In the talk we are going to show that a notion of regularization defined according to what is usually done for ill-posed problems allows to derive learning algorithms which are consistent and provide a fast convergence rate. It turns out that for prior expressed in term of variable Hilbert scales in reproducing kernel Hilbert spaces our results for Tikhonov regularization match those in Smale and Zhou (2005) and improve the results for Landweber iterations obtained in Yao et al. (2005). The remarkable fact is that our analysis shows that the same properties are shared by a large class of learning algorithms which are essentially all the linear regularization schemes. The concept of operator monotone functions turns out to be an important tool for the analysis.

Rates of convergence of Tikhonov regularization for nonlinear inverse problems with stochastic noise

Mihaela Pricop
Institute for Numerical and Applied Mathematics
University of Göttingen
Department of Mathematics
Lotzestr. 16-18
37083 Göttingen
Germany
pricop@math.uni-goettingen.de

In this talk we consider statistical inverse problems described by nonlinear operator equations $F(a) = u$ where a is an element of a Hilbert space and u is an L^2-function. We construct an estimator $\widehat{a}_n$ of a from measurements of u perturbed by random noise using nonlinear Tikhonov regularization in a Hilbert scale setting. We show that this leads to order-optimal rates of convergence for the mean integrated square error $\mathbf{E}\|\widehat{a}_n - a\|^2$ for a range of smoothness classes and without closeness assumptions on the initial guess. Our theoretical results are illustrated by numerical experiments with parameter identification problems for elliptic differential equations.

Risk hull method for inverse problems

Laurent Cavalier
LATP, UMR CNRS 6632
Université Aix-Marseille 1
CMI, 39 rue Joliot-Curie
13453 Marseille cedex 13
France
cavalier@cmi.univ-mrs.fr

We study a standard method of regularization by projections of the linear inverse problem $Y = Af + \epsilon$, where ϵ is a white Gaussian noise, and A is a known compact operator with singular values converging to zero with polynomial decay. The unknown function f is recovered by a projection method using the SVD of A. The bandwidth choice of this projection regularization is governed by a data-driven procedure which is based on the principle of the risk hull minimization. We provide non–asymptotic upper bounds for the mean square risk of this method and we show, in particular, that in numerical simulations, this approach may substantially improve the classical method of unbiased risk estimation.

Calibration of financial Levy models as inverse problem

Markus Reiß
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294
69120 Heidelberg
reiss@statlab.uni-heidelberg.de

A common relatively simple model for financial markets assumes the logarithmic stock price to be a Levy process. Since the market is incomplete, inference for the characteristics of the Levy process must be based on future option prices observed at the market. This calibration problem turns out to be a nonlinear inverse problem where the inverse can be explicitly calculated in the Fourier domain. The proposed estimation method uses a spectral cut-off scheme and attains the minimax rates for an increasing number of observations. The rates correspond to a severely ill-posed problem not only for the nonparametric part, the Levy jump density, but also for the parametric part, the volatility, trend and intensity. In addition to a mathematiocal explanation of these surprising facts we provide some numerical results for simulated and real data.

Nonparametric instrumental variables estimation of a quantile regression model

Joel L. Horowitz
Department of Economics
Northwestern University
Evanston, IL 60208
United States of America
joel-horowitz@northwestern.edu

We consider nonparametric estimation of a regression function that is identified by requiring a specified quantile of the regression error conditional on an instrumental variable to be zero. The resulting estimating equation is a nonlinear integral equation of the first kind, which generates an ill-posed-inverse problem. The integral operator and distribution of the instrumental variable are unknown and must be estimated nonparametrically. We show that the estimator is mean-square consistent, derive its rate of convergence in probability, and give conditions under which this rate is optimal in a minimax sense. The results of Monte Carlo experiments show that the estimator behaves well in finite samples.

Recent results in the modelling of approximation errors in inverse problems

Jari P. Kaipio
Department of Applied Physics
University of Kuopio
PL 1627
70211 Kuopio
Finland
Jari.Kaipio@uku.fi

The errors in the solution of inverse problems that are due to approximation and modelling errors depend on the solution itself. In the deterministic and frequentist paradigms there is little we can do about this. In the Bayesian paradigm we don't know the approximation error either, but we are able to compute its statistical properties. This information can be exploited in the computation of the estimates. We review the theory in the case of linear gaussian models and present the theory for linear nonstationary inverse problems. We show results in the case of estimating thermal coefficients of a nonhomogeneous object.

Convergence rates of general regularization methods for statistical inverse problems

(Joint work with T. Hohage, A. Munk & F. Ruymgaart)

Nicolai Bissantz
Institut für Mathematische Stochastik
Maschmühlenweg 8-10
37073 Göttingen
Germany
bissantz@math.uni-goettingen.de

During the past mean square error convergence analysis for linear statistical inverse regression problems has mainly focused on spectral cut-off and Tikhonov type estimators. Tikhonov estimators have the advantage that they can be implemented easily, however, they achieve minimax rates only in a very restricted number of smoothness classes. In contrast, spectral cut-off estimators achieve minimax rates in a broad range of smoothness classes. However, in order to implement these estimators in practice, the complete spectral information of the operator is required which is often not available. Hence, their applicability is rather limited in practice. In this paper we introduce a unifying technique to study the mean square error of a large class of regularization methods including the aforementioned estimators as well as iterative methods, such as ν-methods and the Landweber iteration. The latter estimators do not require spectral information on the operator but converge at the same rate as spectral cut-off. Convergence rates and numerical performance of these estimators is demonstrated in an example.

Measuring resolution in nonlinear and constrained inverse problems

Philip B. Stark
University of California, Berkeley
Berkeley, CA 94720-3860
United States of America
stark@stat.berkeley.edu

I present a definition of resolution that applies to nonlinear and constrained inverse problems. The definition reproduces the Backus-Gilbert measure of resolution in unconstrained linear inverse problems in Hilbert spaces. The key idea is a constrained optimization problem: find the most localized average that can be estimated with controlled minimax risk.

Joint Emission and Motion Estimation for a Cardiac Cycle in Gated Emission Tomography

Bernard A. Mair
Department of Mathematics
University of Florida
P O Box 118105
358 Little Hall
Gainesville, FL 32611-8105
United States of America
bam@math.ufl.edu

The nuclear imaging techniques (SPECT and PET), based on the decay of an injected radiopharmaceutical are widely used for assessing myocardial viability. This talk discusses the problem of jointly estimating the intensity profiles of the radiotracer and the non-rigid myocardial motion in all stages of the cardiac cycle in gated emission tomography. Previous methods treated these two problems individually. We model the myocardium as an elastic material whose motion does not generate inordinately large amounts of strain. Our proposed method is based on maximizing a penalized likelihood objective function in which the penalty constrains the strain energy of the myocardium in terms of the motion vector field and material parameters. This results in a constrained optimization problem which will be solved by a new two-step iterative method consisting of a modification of Green's one-step late algorithm and a conjugate gradient algorithm. Although the biomechanical penalty depends on the (unknown) elastic properties of the myocardium, the method handles both healthy and diseased hearts.

The algorithm will be presented in a general setting of computing maximum a posteriori (MAP) estimates, so may be of independent interest.

Asymptotic distribution of the MLE in a class of deconvolution models

(Joint work with Stefanie Donauer (VU Amsterdam) and Piet Groeneboom (VU Amsterdam and TU Delft))

Geurt Jongbloed
Faculteit der Exacte Wetenschappen
Vrije Universiteit
De Boelelaan 1081a
1081 HV Amsterdam
Netherland
geurt@cs.vu.nl

Consider a situation that one is interested in the distribution function F of a random variable X. However, no direct sample from F is available. Data only appear via some additional random mechanism. The data are then distributed according to a distribution H_F that depends on F. Deriving the equation expressing H_F in terms of F, can be called the direct problem. Inverting this relation, expressing F in terms of H_F, can be called the inverse problem whereas estimating F based on a sample from H_F furnishes the associated statistical inverse problem. An interesting statistical inverse problem is the so-called deconvolution problem on $[0, \infty)$. We are interested in the distribution function F of X, but only observe X with independent random noise $Y \backsim g$, where g is some (known) probability density on $[0, \infty)$. The observed random variable Z has a density given by

$$h_F(z) = \int_{[0,z]} g(z - x)\, dF(x).$$

Various general nonparametric methods exist that can be used to estimate F based on a sample from h_F. One of these is nonparametric maximum likelihood. The NPMLE $\hat{F}n$ of F is dened as the maximizer (over all distribution functions) of the log likelihood function that is given by

$$\ell(F) = \int \log h_F(z)\, dH_n(z).$$

where H_n denotes the empirical distribution function of the observed Z values. If g is the standard exponential density, $\hat{F}_n$ can be computed explicitly and its asymptotic distribution follows from its explicit characterization. For more general decreasing densities g, this is not the case and only im- plicit characterizing (in-)equalities are available. We employ a method to derive the asymptotic distribution of $\hat{F}_n(x_0)$ based on the asymptotic anal- ysis of these necessary and suffcient optimality conditions. This method is very promising and we expect it can be used to derive asymptotic theory in many other interesting estimation problems where an explicit expression for the estimator of interest is lacking.

Jump reconstruction in certain inverse problems

Leif Boysen
Institut für Mathematische Stochastik
Maschmühlenweg 8-10
37073 Göttingen
Germany
boysen@math.uni-goettingen.de

We study change-point estimation from indirect noisy observations of a step function. Contrary to the case where the objective function is in a non-parametric class, the rates for estimating the change-point(s) are not sensitive to the degree of ill-posedness of the problem. To be more precise, the change-points and jump heights of the objective function can be estimated at a square-root of n rate for a large class of convolution kernels, including supersmooth functions as the Gauss kernel as well as polynomial smooth functions as the Laplace kernel. We show asymptotic normality of the estimates. Unlike in the direct case, the estimators are no longer asymptotically independent. A special case of the model is the so called multi-phase or two-phase linear regression with unknown break points.

Estimation of a convex density: Back to Hampel birds problem

Fadoua Balabdaoui
Institut für Mathematische Stochastik
Maschmühlenweg 8-10
37073 Göttingen
Germany
fadoua@math.uni-goettingen.de

Motivated by Hampel birds migration problem, Groeneboom, Jongbloed and Wellner (2001b) established the asymptotic distribution theory for non-parametric Least Squares and Maximum Likelihood estimators of a convex and decreasing density at a fixed point $x_0 > 0$. However, in the inverse statistical problem of interest, estimation of the distribution function of the birds resting time depends on the value of the first derivative of the estimators at 0, at which they are not consistent.

In this paper, we focus on the Least Squares estimator and show that it is possible, following the idea of Kulikov and Lopuhaä (2005) in monotone estimation, to construct a consistent estimators as long we stay away from 0. Moreover, we derive their exact asymptotic distributions.

Sharp Adaptation for Statistical Inverse Problems on Manifolds

Peter Kim
University of Guelph
Department of Mathematics & Statistics
50 Stone Road East
Room 536 MacNaughton Building
Guelph ON Canada N1G 2W1
Canada
pkim@uoguelph.ca

This talk will examine the estimation of an indirect signal embedded in white noise over a compact manifold. It will be shown that the sharp minimax bound is determined by the degree to which the indirect signal is embedded in the linear operator. Thus when the linear operator has polynomial decay, recovery of the signal is polynomial, whereas if the linear operator has exponential decay, recovery of the signal is logarithmic. The constants are determined for both of these classes and adaptive sharp estimation is also carried out. In the polynomial case a blockwise shrinkage estimator is needed while in the exponential case, a straight projection estimator will suffice. Some of the results depend on aspects of spectral geometry and in particular, the asymptotic eigenvalue calculations associated with H. Weyl.

Instrumental regression in partially linear models

(Joint work with Jan Johannes & Sébastien Van Bellegem)

Jean-Pierre Florens
Université Toulouse 1, Sciences Sociales
Place Anatole
31042 Toulouse Cedex
France
florens@cict.fr

We consider the semiparametric regression $X^t\beta + \phi(Z)$ where β and $\phi(\cdot)$ are unknown slope coefficient vector and function, and where the variables (X, Z) are endogeneous. We propose necessary and sufficient conditions for the identification of the parameters in the presence of instrumental variables. We also focus on the estimation of β. An incorrect parametrization of ϕ generally leads to an inconsistent estimator of β, whereas consistent nonparametric estimators for β have a slow rate of convergence. An additional complication is that the solution of the equation necessitates the inversion of a compact operator which can be estimated nonparametrically. In general this inversion is not stable, thus the estimation of β is ill-posed. In this paper, a $\sqrt{n}$-consistent estimator for β is derived under mild assumptions. One of these assumptions is given by the socalled source condition which we explicit and interpret in the paper. Finally we show that the estimator achieves the semiparametric efficiency bound, even if the model is heteroskedastic.

Kernel density estimation for the coefficients in random coefficient regression with applications to demand analysis

(Joint work with Stefan Hoderlein and Jussi Klemelä)

Enno Mammen
University of Mannheim
Department of Economics
68131 Mannheim
Germany
emammen@rumms.uni-mannheim.de

We consider a linear regression model where the coefficients are random. We propose a new estimator for the probability density of the coefficients. The estimator is based on kernel smoothing. We derive the rate of convergence of the estimator in the L_2 and pointwise loss and show that it achieves optimal rates in smoothness classes. The kernel estimator is related to estimation problems in tomography and it is based on an empirical inverse Radon transform. The paper is motivated by an econometric application in demand analysis.

Passive detection and imaging of nuclear material using cosmic ray muons

Nick Hengartner
D-1 Statistical Science Group
Los Alamos National Laboratory
United States of America
nickh@lanl.gov

Existing radiographic methods are inefficient for detecting shielded nuclear materials and potentially presents radiation hazards to inspectors and vehicle passengers. Recent advances at the Los Alamos National Laboratory suggest a promising alternative: To use the natural scattering of muons – produced by the decay of cosmic rays showering down on Earth – as a passive radiographic probe to image dense objects. Muons interact with matter primarily through the Coulomb force. We model the net effect of that interaction as a random changes in the path of the muon, with the magnitude of that change depending on the atomic number of the material traversed. In this talk, I discuss the tomographic reconstruction of the scattering density of an object from measured changes in the paths of individual muons traversing that object. This problem is challenging in a number of ways: the signal of interest is mainly contained in the variance, the geometry of the detectors impacts what features of the scattering density can recovered, and the difficulty of covariate measurements (such as the momentum of each muon) that also impacts the magnitude of the scattering. In this talk, I discuss both estimation and a approach to analyze an experimental set-up prior to the experience.

Poster

Large scale statistical parameter estimation in complex systems with an application to metabolic models

Daniela Calvetti
Case University
10900 Euclid Avenue
Cleveland, OH 44106
United States of America
daniela.calvetti@case.edu

The estimation of a large number of parameters in a complex dynamic multi- compartment model in the presence of insufficient data is a difficult and chal- lenging problem. Such problems arise in many applications e.g. in biology, physiology and environmental sciences. The model consists of a large system of coupled non-linear ordinary differential equations, the data consisting of the values of few components at given observation times. The estimation prob- lems are usually ill-posed and severely underdetermined, while the quality of the scarce data is far from optimal. Therefore, a successful solution necessar- ily requires additional information about the parameters. A natural framework to introduce a priori information into the model is the Bayesian paradigm. We propose a Bayesian methodology that is able to utilize various types of prior con- straints such as approximate algebraic constraints for the parameters, inequality constraints for the solutions or belief about the dynamic response of certain en- zymes and integrate them into a parametric prior distribution. The subsequent parameter estimation is based on a combination of optimization methods and statistical sampling techniques. We apply the methodology to a cardiac muscle metabolism model, where we are able to simultaneously estimate more than one hundred parameters from one tenth as many measured data points.

An inverse problem in superconductivity

Harald Heese
Institute for Numerical and Applied Mathematics
University of Göttingen
Department of Mathematics
Lotzestr. 16-18
37083 Göttingen
Germany
heese@math.uni-goettingen.de

Current transport through thin superconducting films in the flux-free Meissner state can be described adequately in terms of potential theory and boundary integral equations. The current distributions mainly suffer from inhomogeneities due to self-enhanced shielding currents. In the mathematical model the current distribution corresponds to the jump

$$\left[\frac{\partial u}{\partial \nu_\Gamma}\right] := \frac{\partial u_-}{\partial \nu_\Gamma} - \frac{\partial u_+}{\partial \nu_\Gamma}$$

of the normal derivative of the scalar magnetic potential u across the superconducting film Γ. The shielding currents may be influenced by placing permanent magnets near the film, which can be realized in the same model as additional transmission boundaries.

For applicational purposes a homogenization of the current distribution is required. This leads to an operator equation of the form

$$F(\gamma) = \text{const} \qquad (*)$$

which needs to be solved for the free transmission boundaries γ, where F denotes the mapping from the transmission boundaries to the associated current distribution on the film Γ. It turns out that $(*)$ is ill-posed since the range of F does not comprise the constants. Hence, this inverse problem is recast as a geometric optimization problem for the weighted least squares functional

$$J(\gamma) := \frac{1}{2}\int_\Gamma w(x)\Big[\Big(F(\gamma)\Big)(x) - \text{const}\Big]^2\, ds(x).$$

We solve the optimization problem numerically presenting examples for two different approaches. The first proceeds directly along the lines of Fréchet differentiability of boundary integral operators, whereas the second seeks to employ the level set methodology.

A new Framework for Assessing Uncertainty in Ill-Posed Problems

Andreas Hofinger
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
Altenbergerstrasse 69, Hochschulfondsgebäude 110
A-4040 Linz,
Austria
andreas.hofinger@oeaw.ac.at

The metrics of Prokhorov and Ky Fan provide a well-suited framework to quantify convergence speeds for stochastic ill-posed problems: The new framework is applicable to a wide class of problems, in particular it also works in cases where the expected value diverges. Furthermore, existing results like convergence rate results of the deterministic theory or probability estimates of the form $\mathbb{P}(x > \varepsilon) < \delta$ can be transferred to the new setup easily. Finally, the Bayesian approach can be treated as well, since the Prokhorov metric intrinsically deals with distributions. The new framework therefore also allows in the Bayesian approach to answer questions concerning convergence and convergence rates.

Nonlinear Integral Equations in Inverse Obstacle Scattering

(Joint work with R. Kress)

Olha Ivanyshyn
Institute for Numerical and Applied Mathematics
University of Göttingen
Department of Mathematics
Lotzestr. 16-18
37083 Göttingen
Germany
ivanyshy@math.uni-goettingen.de

We consider the inverse problem of time-harmonic acoustic wave scattering to reconstruct the shape of an obstacle from a given incident field and the modulus of the far field pattern. For this problem we use a novel solution method based on a pair of nonlinear and ill-posed integral equations for the unknown boundary that arises from the reciprocity gap principle. The integral equations can be solved by linearization, i.e., by regularized Newton iterations.

Particle Spectra by the Application of Regularization Methods

(Joint work with E. Böhm and R. Wimmer-Schweingruber)

Alexander Kharytonov
Institute of Experimental and Applied Physics
Kiel University
Olshausenstrasse 40
D-24098 Kiel
Germany
exp503@physik.uni-kiel.de

The interpretation of experimental data is often achieved by the method of deconvolution, leading to the Fredholm integral equations of the first kind. The numerical solution of integral equations reduces to ill-conditioned or singular linear systems. To solve these algebraic equations, regularization methods such as the singular value decomposition (SVD) or the Tikhonov methods are available. Linear inequality constraints may complicate the analysis and the multiple regularization is a method to solve those inverse problems. It consists of solving at first the algebraic problem and then the minimization problem.This method is applied to the experimental data from the EPHIN-sensor on the satellite SOHO to derive the energy spectrum of solar particles. The matrices for the algebraic problem have been calculated by Monte Carlo methods. For this the GEANT4-package of the CERN-program-library has been used to simulate the detector response to nuclear particles in the EPHIN-sensor.

Efficient tests for the deconvolution hypothesis

(Joint work with A. Munk)

Michael Langovoy
Institut für Mathematische Stochastik
Maschmühlenweg 8-10
37073 Göttingen
Germany
langovoy@math.uni-goettingen.de

We constructed new goodness of fit tests for statistical inverse problems. Tests are designed to be asymptotically efficient. Tests are capable to choose the best model dimension automatically by the data.

Non-convex regularization

(Joint work with Markus Grasmair and Otmar Scherzer)

Frank Lenzen
Institut für Informatik
Universität Innsbruck
Technikerstr. 21a
6020 Innsbruck
Austria
frank.lenzen@uibk.ac.at

For the inverse problem of denoising the common approach is to use Tikhonov regularization. For denoising of discontinuous data the total variation regularization was developed. In contrast to classical Tikhonov regularization, the functional is convex but not differentiable. The next step toward generalization of regularization methods is non-convex regularization.

We motivate non-convex regularization models from Bayesian statistics and sampling theory, present some preliminary analysis and support the results by numerical experiments.

Statistical Inverse Problems on the Euclidean Motion Group

Maia Lesosky
University of Guelph
Department of Mathematics & Statistics
50 Stone Road East
Guelph ON Canada N1G 2W1
Canada
mlesosky@uoguelph.ca

The estimation of an indirect signal embedded in white noise on the Euclidean Motion Group, SE(2), is investigated. The linear minimax risk is calculated and related to the overall minimax risk.

Sampling with Finite Fourier and Hankel Transform Eigenfunctions

Tatiana Levitina
Institut Computational Mathematics
Technische Universität Braunschweig
Pockelsstraße 14
D-38106 Braunschweig
Germany
t.levitina@tu-bs.de

As shown in recent papers [1,2], for band-limited functions mainly concentrated on some time interval, the truncation error of sampling expansions with Finite Fourier Transform Eigenfunctions (FFTE) is essentially less than that of Shannon's series.

This allows multiple applications, in particular, when the FFTE are used as filtering functions in the Filter Diagonalization procedure [3]. The latter serves to extract spectral information from a short-time signal at a desired energy range [4].

Sampling series similar to the Walter and Shen FFTE expansions are obtained for Hankel–band–limited functions in terms of Finite Hankel Transform Eigenfunctions(FHTE); they might be especially important in, e.g., computational tomography. On the efficient and accurate numerical methods for both FFTE and FHTE evaluation, as well as for computation of integrals containing these functions, we refer to [5,6].

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Regularization of inverse problems with noisy operator

Clément Marteau
CMI, 39 rue F. Joliot-Curie
13 453 Marseille cedex 13
France
clement.marteau@cmi.univ-mrs.fr

We study statistical inverse problem when the operator is not well-known. Using the penalized blockwise Stein's rule, we construct an estimator that produces sharp asymptotic oracle inequalities in different settings. In particular, we consider the case where the set of bases is not associated to the singular value decomposition. The representation matrix of the operator is not diagonal and the regularization problem becomes more difficult."

Stochastic spectral methods for Bayesian inference in inverse problems

(Joint work with Habib N. Najm)

Youssef M. Marzouk
Sandia National Laboratories
Livermore, CA 94551
United States of America
ymarzou@sandia.gov

The Bayesian setting for inverse problems provides a foundation for inference from noisy data and uncertain forward models, a natural mechanism for incorporating prior information, and a quantitative assessment of uncertainty in the inferred results. Obtaining useful information from the posterior density—e.g., via Markov chain Monte Carlo (MCMC)—may be a computationally expensive undertaking, however. For complex and high-dimensional forward models, such as those that arise in inverting systems of PDEs, the cost of likelihood evaluations may render Monte Carlo simulation prohibitive.

This work seeks to accelerate Bayesian inference by introducing polynomial chaos (PC) expansions for unknown model parameters. The PC construction employs orthogonal polynomials in i.i.d. random variables as a basis for the space of square-integrable random variables. Given uncertain inputs that span the range of the prior, a Galerkin solution of the resulting stochastic forward problem with this basis yields a PC expansion for uncertain forward model predictions. Evaluation of integrals over the parameter space is then recast as Monte Carlo sampling of the random variables underlying the PC expansion.

We evaluate the utility of this technique on a transient diffusion problem arising in contaminant source inversion. The accuracy of posterior estimates is examined with respect to the order of the PC representation, the choice of PC basis, and the decomposition of the support of the prior. The computational cost of the new scheme shows significant gains over direct sampling.

On Deconvoluting Densities

Monika Meise
Universität Duisburg-Essen, Campus Essen
Fachbereich Mathematik
Universitätsstraße 2
45117 Essen
Germany
monika.meise@uni-due.de

Given data $y_1, \dots, y_n$ we look for an approximating model of the form

$$Y_i = X_i + Z_i, \quad i = 1, \dots, n \qquad (*)$$

where the $(X_i)_1^n$ and $(Z_i)_1^n$ are respectively i.i.d random variables and the distribution of the $(Z_i)_1^n$ is given. The problem is to decide whether there exists an approximation of the form $(*)$ and, if so, to specify a distribution of the $(X_i)_1^n$. This can be done by choosing the distribution of X so as to minimize the Kolmogorov distance $d_{ko}(F_n, F^Y)$ where F_n is the empirical distribution of the data and F^Y the distribution of the random variables Y. Variations include the Kuiper distance $d_{ku}(F_n, F^Y)$ and minimizing the total variation of the density f^X of X or its first or second derivative.

Regularized Fixed-Point Iteration for Nonlinear Inverse Problems

(Joint work with R. Pinnau and N. Siedow)

S. S. Pereverzyev
Fraunhofer-Institut für Techno- und Wirtschaftsmathematik
Gottlieb-Daimler-Straße
D-67663 Kaiserslautern
Germany
pereverz@itwm.fhg.de

Let us consider a nonlinear inverse problem given by the operator equation $Fu = y$, which we assume to have a solution $\hat{u}$ for the noise-free data y. We would like to present a new derivative-free method for approximating $\hat{u}$. It is based on the representation of the nonlinear operator F as the sum of some linear operator A and nonlinear operator G. This suggests the following iterative procedure for solving the inverse problem: $Au_{k+1} = y - Gu_k$.

In the case of ill-posed problems, we consider the situation when both the original nonlinear operator F and the linear operator A do not have bounded inverse. That's why, when dealing with the noisy data y_δ, $\|y_\delta - y\| \leq \delta$, each iteration in the proposed procedure needs to be regularized.

We present conditions on the operators A and G that guarantee monotonic decrease of the iteration error $\|\hat{u} - u_k\|$ to some level $b(\delta)$. If one continues the iterative process, than the error remains below $b(\delta)$. It has an order-optimal value with respect to the noise level δ provided that the solution $\hat{u}$ satisfies the general source condition.

We give several examples of the inverse problems where suggested procedure is successfully applied.

Detecting corrosion using thermal waves

(joint work with T. Hohage and F.-J. Sayas)

M.-L. Rapún
Dep. Matemática Aplicada (Biomatemática)
Universidad Complutense de Madrid
Ciudad Universitaria
28040 Madrid, Spain
mlrapun@bio.ucm.es

Nondestructive obstacle detection through low frequency wave propagation motivates several mathematical inverse problems with applications in industry, such as radar or medical imaging. Among this class of problems, we are interested in solving the inverse acoustic scattering problem for both the shape and the impedance, given the far-field pattern for one single incident plane wave.

In this work, we suggest a hybrid method to numerically solve this inverse problem. On one hand, this method is an iterative method that at each step updates the approximations for both the unknown boundary of the obstacle and the unknown impedance. On the other hand, each iteration step is divided in two sub-steps (one ill-posed and the other nonlinear) in the spirit of the Kirsch-Kress decomposition method. Therefore the method can be seen as a hybrid between iterative and decomposition methods, inheriting advantages from each of them, such as obtaining good reconstructions and not needing a forward solver at each step. Numerical results are presented showing the feasibility of the proposed method.

A hybrid method for inverse scattering for shape and impedance

Pedro Serranho
Institut für Numerische und Angewandte Mathematik
Georg-August Universität
Lotzestrasse 16-18,
37083 Göttingen
Germany
serranho@math.uni-goettingen.de

Nondestructive obstacle detection through low frequency wave propagation motivates several mathematical inverse problems with applications in industry, such as radar or medical imaging. Among this class of problems, we are interested in solving the inverse acoustic scattering problem for both the shape and the impedance, given the far-field pattern for one single incident plane wave.

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Data approximation and inverse problems

Henrike Weinert and Thoralf Mildenberger
Department of Statistics, SFB 475
University of Dortmund
Germany
weinert@statistik.uni-dortmund.de
mildenbe@statistik.uni-dortmund.de

The principle of data approximation is a new approach to model choice. Its aim is to choose an appropriate model based on particular features of the data which the model is required to reproduce. Such features depend on the field of application and should be specified according to the subject matter under investigation. In simulation studies, so called residual based methods have given very good results for nonparametric regression problems: candidate regression functions of increasing complexity are produced until the residuals look like white noise. As a first application we use a residual based technique for a parametric deconvolution problem, where the object of interest is known to consist of delta spikes.

The evolution-observation scheme in Blagovestchenskii's approach to the 1D inversion

(Joint work with A.P.Katchalov and Y.V.Kurylev)

Victor Zalipaev
Department of Mathematical Studies
University of Loughborough
Loughborough LE11 3TU
United Kingdom
V.Zalipaev@lboro.ac.uk

In this paper we combine the evolution-observation scheme [1] with solving the 1D inverse problem of wave propagation into inhomogeneous half-space by means of Blagovestchenskii's approach. Initially, in the deterministic part the dynamical inverse problem is studied in the acoustical approximation with a point boundary source. The velocity profile is assumed to be dependent only on depth coordinate, z. It is reduced to a system of non-linear Volterra type integral equations with additional dependence on $|\xi|$ parameter. It appears due to Fourier transform $x \Leftrightarrow \xi$ where $x = (x_1, x_2)$ are horizontal coordinates. The reconstruction of the velocity uses the response function $r(t, |\xi|)$, where t is the time variable. This leads to overdeterminancy of inverse data with respect to $|\xi|$. This makes possible to apply the evolution-observation scheme (Kalman filter) with respect to $|\xi|$ to the inverse problem. To this end we linearize the non-linear Volterra type system of integral equations and use the standard evolution-observation techniques updating the conditional probability distribution of the estimated error in the velocity reconstruction.

[1] Kaipio, J., Somersalo, E., *Statistical and Computational Inverse Problems*, Applied mathematical sciences, **160**. Springer-Verlag, Berlin, 2005.

Organization Committee:

Frank Bauer, Nicolai Bissantz, Thorsten Hohage and Axel Munk

Financial Support:

Deutsche Forschungsgemeinschaft DFG

Graduiertenkolleg 1023,
Identification in Mathematical Models:
Synergy of Stochastic and Numerical Methods

DAAD Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

Sponsoring:

Taylor & Francis
Taylor & Francis Group

Paperwork:

Heike Ahrens, Frank Bauer and Eric Heinemeyer

Other Support:

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